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# On the resonant nature of the breakdown of a laminar boundary layer

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The first part of this paper (§2) briefly reviews the history of the idea of the resonant nature of laminar-boundary-layer breakdown. In the second part a new wave-resonance concept of the breakdown mechanism is proposed. The existing experimental data on the laminar boundary layer (and plane channel flow) breakdown are analysed from the viewpoint of this concept and are compared with the well-known local high-frequency secondary-instability concept. The results testify to the correctness of the proposed wave-resonant concept for the initial stages of breakdown, in particular for the K-regime of transition up to the spike formation and its doubling.

Within the framework of the wave-resonance concept, before constructing the corresponding theory, many important features of the disturbance development can be qualitatively explained and understood. Concerning the understanding of the spike appearance, the wave-resonance concept complements the local high-frequency secondary-instability one and represents by itself a new fruitful viewpoint on this phenomenon. The development of the wave-resonance concept and its application to the analysis of numerical and physical experiments, together with the construction on this basis of the corresponding theory, can give an essential impetus towards the better understanding of the breakdown nature.

## 1. Introduction

The progress of hydrodynamic stability theory and turbulence-onset studies has led to the understanding that transition starts long before the pronounced phenomena of breakdown are seen, in the laminar flow in the vicinity of the leading edge of a model, and even earlier in the external free stream. The onset of turbulence in the boundary layer comprises three main stages, schematically given in the book by Kachanov, Kozlov & Levchenko (1982), which correspond to the three main aspects of the problem that are studied theoretically as well as experimentally.

At the first stage, in the region of small Reynolds numbers (from the plate nose up to the first branch of the neutral stability curve), the generation of the instability waves (i.e. boundary-layer eigenoscillations, usually called Tollmien–Schlichting waves) takes place. The problem of the generation of these waves by disturbances of different kinds (acoustic, vortical and vibrational external disturbances) is often called the problem of boundary-layer receptivity to external disturbances. This was clearly formulated for the first time by Morkovin (1968) more than 15 years ago, but only recently have the first successes in its solution been achieved (see Kachanov *et al.* 1982). In particular, the important role of the model vibration in the problem of receptivity was first experimentally shown and studied by Kachanov, Kozlov &

Levchenko (1975), and one of the mechanisms of the transformation of free-stream vortices into Tollmien–Schlichting waves (the mechanism of the leading edge) was first analysed by the same authors (1978*b*). Studies of the excitation of instability waves responsible for the transition to turbulence have recently been intensively carried out, especially in the USSR and the USA. To understand the current state of this problem as well as the problem of turbulence onset in general, one must turn to the proceedings of the *IUTAM Symposium on Laminar–Turbulent Transition* which was held in Novosibirsk, July 1984 (see Kozlov 1985).

The second stage of the transition corresponds to the propagation of instability waves of small amplitude down the boundary layer and to their amplification, if the flow is unstable to them, or otherwise to their attenuation. This stage is described by the linear hydrodynamic stability theory and has been thoroughly studied at least for the case of two-dimensional flows. Since this stage is quite extensive and also since the phenomena that take place in the linear region are simply described, the linear theory of stability together with receptivity theory are very important for the construction of engineering methods of transition calculation based on accurate physical understanding of this phenomenon (see Kachanov *et al.* 1982).

Finally, when the amplitudes of unstable Tollmien–Schlichting waves reach considerable values (1–2 % of the flow velocity) the flow enters a phase of nonlinear breakdown, randomization and a final transition into a turbulent state. This breakdown phase is not very extensive, but here transformation of a deterministic, regular, often two-dimensional laminar flow into a stochastic and at the same time regular, three-dimensional, yet mysterious turbulence takes place. Although the region of nonlinear breakdown has been studied for about 40 years, many aspects are still unknown. Experimental and theoretical models are being built, hypotheses are being made, but the advance is slow.

But in recent years, owing to the joint efforts of specialists from different countries (primarily the USSR, Great Britain and the USA as well as Japan and FRG), considerable progress in the understanding of the dominant mechanisms of the process of breakdown and randomization of laminar flow has been achieved. This progress is associated with the discovery of the important (or even decisive) role of resonant phenomena which occur in the process of transition and which determine the flow breakdown. This idea, which arose from a series of experimental and theoretical works, let us think that the process of laminar-boundary-layer breakdown of different types is of a resonance nature.

The studies of this field are now in full swing. Many questions are not yet clearly answered, many hypotheses are still to be proved. The purpose of this paper is to briefly review the history of the birth and formation of the idea of the resonant nature of laminar-boundary-layer breakdown, to formulate a new, wave-resonance concept of breakdown and to verify it using the available experimental data.

## 2. Two types of breakdown: role of resonant interaction

### 2.1. *N-type of transition. Discovery of its resonant nature*

The first studies of the laminar-flow breakdown in the boundary layer were carried out by Schubauer & Scramsted (1947). This work served as an experimental basis for the concept of hydrodynamic instability and gave the first information about nonlinear mechanisms of transition. At the end of the 1950s the fundamental experiments, which later became classical, were carried out by Klebanoff, Tidstrom & Sargent (1962), that laid the foundation of many modern ideas about laminar-boundary-layer breakdown. In these experiments controlled disturbances simulated the 'natural' ones observed by Schubauer & Klebanoff (1956) and by Klebanoff & Tidstrom (1959). In the work of Klebanoff *et al.* (1962) the mechanisms of the laminar-flow breakdown were studied in detail and the applicability of a series of theoretical models and hypotheses existing at that time was critically estimated. New important features of the nonlinear breakdown were identified. A detailed investigation of the three-dimensional-flow velocity field in the same regime of the transition was carried out by Kovasznay, Komoda & Vasudeva (1962).

In particular, Klebanoff *et al.* (1962) discovered that the breakdown starts with the appearance on the oscilloscope traces of powerful high-frequency flashes or 'spikes' which are doubled, tripled etc. downstream. (Later, Kachanov *et al.* 1984 showed that single-spike oscilloscope traces are observed in double-, triple- etc. spike stages too. They exist further from the wall.) It could be concluded that these flashes–spikes generate turbulent spots, which are responsible for the onset of intermittency. The appearance of the spikes was attributed to the local high-frequency secondary instability of the primary wave due to inflexion points in the instantaneous profiles of flow velocity. These notions about the role of spikes in the laminar-flow breakdown as well as about the causes of their appearance were predominant for more than two dozen years and are popular now. Up to the middle of the 1970s the general opinion was that the succession of nonlinear phenomena discovered in the experiments by Klebanoff *et al.* (1962), which lead to the onset of the turbulent regime, is a fundamental property of the flow in a boundary layer. The results obtained in that work served as a basis for the majority of further experimental as well as theoretical studies, either specifying the data obtained by Klebanoff *et al.* (1962) and Kovasznay *et al.* (1962) or attempting to explain phenomena described in these papers. The Klebanoff period of investigations ended in 1976.

In the work by Kachanov, Kozlov & Levchenko (1977, hereinafter referred to as I) in Novosibirsk experimental data were obtained that testify to the existence of a new type of laminar-flow breakdown in a boundary layer (N-breakdown). In particular, in a newly discovered transition regime (which is now often lamely called 'subharmonic') no flashes of high-frequency pulsations (spikes), typical of the Klebanoff breakdown (K-breakdown), were observed and neither were intermittency or localized turbulent spots. The transition was realized through the gradual growth of higher harmonics, the appearance of low-frequency oscillations in a spectrum, including a subharmonic, and their successive interaction with high-frequency oscillations which smoothly fill up the spectrum. In I it was noted that the onset of low-frequency oscillations in the spectrum was always accompanied by three-dimensionality and served as a starting device for the beginning of the breakdown and randomization of the laminar flow. Later, in the experiments by Saric, Carter & Reynolds (1981) and Thomas & Saric (1981) carried out simultaneously with the work of Kachanov & Levchenko (1982, hereinafter referred to as II) the onset of three-dimensional subharmonics (a chess-board pattern of $\Lambda$-vortices with frequency $\frac{1}{2}f_1$) was discovered as in I, but by means of visualization. These works, carried out in other installations, confirmed the role of the subharmonic in the N-transition of the boundary layer observed in I.

A study of the causes of the excitation of the low-frequency-oscillation packet and of the reasons why the N-type breakdown differs so greatly from the classical K-regime of breakdown, was carried out in II. The same N-regime of transition described in I, five years previously, was reproduced in these experiments. The phase structure of disturbances in a narrow bandwidth of a frequency filter was studied.

The parametric resonant interaction, which amplifies subharmonic priming (or initial) disturbances, proved to be the principal mechanism responsible for the flow randomization in the new N-type of breakdown. A detailed study of the properties of the observed resonance was also carried out in II.

The resonant mechanism of the subharmonic amplification was first theoretically proposed by Raetz (1959) and further studied for the boundary-layer case by Craik (1971) and later by Craik (1978), Volodin & Zelman (1978), Zelman & Maslennikova (1982) and others. But until 1980 the resonance had not been observed experimentally. As a result, many specialists started to doubt whether the resonant triads could play a significant role in boundary-layer transition. The boundary-layer nonparallelism and a continuous changing of the Reynolds number seemed to break up a fragile resonance, the existence conditions of which were strongly influenced by the presence of dispersion for the boundary-layer waves. But, in II the resonance was found. Craik and others studying the resonant triads had not paid particular attention to the phase relationship of the waves and to the important property of parametric resonances of amplifying only those disturbances with definite phases. But it is this property that allowed the discovery of the resonance experimentally and determined the important features of the breakdown.

It was shown experimentally in II that there is a large resonance width involving a wide continuous spectrum of priming oscillations. As a result, a deterministic resonance mechanism in a real transition promotes the growth of a wide bandwidth of low-frequency oscillations with a continuous spectrum. It was also shown that this spectrum of resonantly excited waves must possess an approximate symmetry with respect to the subharmonic of half the fundamental frequency, but need not necessarily have its maximum in the region of the subharmonic. These properties typify those observed in the N-regime. In the appendix of II it was shown that the amplification of a packet of low-frequency oscillations can be regarded as both three-wave resonance (from a quasi-stationary viewpoint) and multi-wave resonance (from a stationary viewpoint). Therefore it can be said that the existence of multi-wave resonances was shown experimentally in II before they were studied theoretically by Zelman & Maslennikova (1982).

The rapid resonance growth of disturbances found in II, despite some differences, is in good agreement with the theory of Zelman & Maslennikova (1982). Their calculations numerically demonstrated the properties of the resonant sets of waves that were discovered experimentally in II. In particular, a large spectral resonance width was demonstrated as well as the property of symmetrization of resonantly amplified harmonics. Other characteristic features of three- and multi-wave resonance were demonstrated in the theory. In particular, as far back as in the work by Volodin & Zelman (1978) it was shown that the amplitude of subharmonics grows in triplets in the double exponent – a property well-known for the parametrical resonance in oscillatory sytems.

As already noted, after the work in I the subharmonic pulsations were observed in the flow-visualization experiments of Thomas & Saric (1981) and Saric *et al.* (1981). As in I and II, in these experimental investigations it was concluded that the N-type of transition of a laminar boundary layer occurs at comparatively small initial amplitudes of the introduced instability wave. The visual structure of the chess-board pattern of $\Lambda$-vortices obtained by Thomas & Saric (1981) and by Saric *et al.* (1981) corresponds to the manifestation of the subharmonics that have attained a larger amplitude. One can also show that the chess-board pattern of $\Lambda$-vortices qualitatively corresponds to the modulation in $z$ of the amplitudes and phases of a subharmonic found in II, and this indicates the amplification of a pair of subharmonic waves inclined at opposite angles to the flow direction. The angles of their wavenumber vectors relative to the flow direction were about $\pm 60°$, as in the experiments in II, and differ a little from the theoretical ones.

The work II was followed by the investigations of Kozlov & Ramazanov (1984) and of Saric, Kozlov & Levchenko (1984) who used almost the same technique. In the former work the amplification of subharmonics was observed in a plane Poiseuille flow and, in the latter, combined hot-wire and visual studies were carried out. Some of the conclusions of II were confirmed and the range of measurements was enlarged up to the final transition to turbulence. It was shown also how an N-type of breakdown can be transformed into K-type one and vice versa depending on the initial conditions.

In the theoretical work by Herbert (1983) a theory of the parametric resonance, which differed a little from Craik's (1971) resonance, was created that studied subharmonic wave amplification in plane channel flow. In this model of the resonance, subharmonics are present – Squire waves – that always have vorticity normal to the wall and a $v$-component of oscillation identically equal to zero. The characteristic feature of these waves is that they have no dispersion about the angle of propagation, and consequently, they should have a considerably weaker angular selectivity when the resonance occurs. The selectivity is connected only with the dependence on the angle of the coefficients of the intermode coupling (Herbert 1983). The application of this theory to the boundary layer (Herbert 1984) gave results that are in good agreement with the experimental data of II. The question as to which theory (that of Craik or that of Herbert) corresponds with the resonance found in the experimental work II is not yet clear.

Numerous recent numerical investigations by Kleiser (1982), Orszag & Patera (1983), Wray & Hussaini (1984), Laurien (1986) and others demonstrate a good agreement with experiments. It was shown that direct computations, which use in particular spectral methods, give much additional information about breakdown processes in the boundary layers and channel flow. On the other hand, these numerical experiments, as well as physical ones, by themselves do not ensure a deeper understanding of the transition process and need physical interpretation. The union of numerical methods and correct physical concepts is an effective way towards successful progress in this field.

### 2.2. *Re-estimation of the nature of* K-*breakdown*

There were many questions to be clarified concerning the classical K-regime of breakdown, the study of which began with the work by Klebanoff *et al.* (1962). Despite a large number of theoretical models that tried to explain the various phenomena, many fundamental questions were not clear: for example, what was the cause of the onset of strong three-dimensionality with a definite preferred period of the spanwise modulation along $z$? Another important question was the cause of the spike generation. There were insufficient data to establish the correctness of any one model. Many problems arose because the language of the analysis given by Klebanoff *et al.* (1962) concerned local wave properties in space and time, and this differed greatly from the wave-spectral notions usually used in the theories.

In 1980 in Novosibirsk detailed studies of the K-regime of breakdown were conducted under more regular controlled conditions employing the technique of frequency and frequency–wavelength complex Fourier analysis of the data. Part of the results of the measurements was presented in the paper by Kachanov *et al.* (1984).

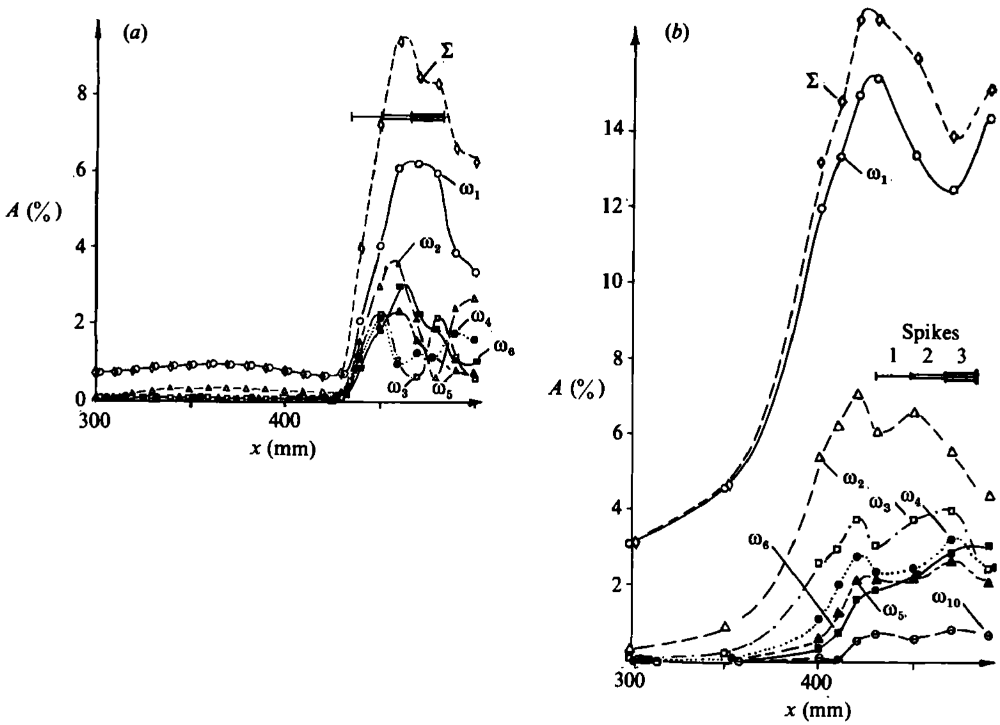


FIGURE 1. Amplification curves of harmonic amplitudes (from Kachanov *et al.* 1984) measured at (*a*) the peak position at $y = 1$ mm = const. and (*b*) for the maxima in $y$-profiles of each harmonic. $\Sigma$ is integral oscillations.

The results obtained radically changed existing notions about the mechanisms of breakdown in the K-regime.

In this work all the manifestations of the K-regime found by Klebanoff *et al.* (1962) and later investigators were reproduced, though some flow and disturbance parameters were somewhat changed. The frequency analysis of the oscillations showed that the spectrum, up to the stage of appearance of the developed spikes, contains only the high harmonics $nf_1$ (up to $n \approx 20$–$50$). The first important conclusion of the work by Kachanov *et al.* (1984) was that the flashes–spikes are not of a stochastic character, but strictly deterministic, periodic and correspond to the process of generation in the spectrum of higher harmonics of the fundamental wave. (It should be noted that the role of higher harmonics was underestimated in the work by Klebanoff *et al.* 1962.) In the K-regime the appearance of spikes does not lead to the randomization of the flow; this process occurs at subsequent stages. Thus, the question of the nature of the randomization of the flow in the K-regime still remains open.

The growth curves of the amplitudes of the harmonics downstream are presented in figure 1(*a*), and it would seem that they demonstrate the explosion of high-frequency-harmonic amplitudes in the region of the spike formation. But, first, the amplitudes of all the harmonics increase (not only of the high-frequency ones). Secondly, the effect of the explosion, as shown by Kachanov *et al.* (1984) is connected with the evolution of oscillation $y$-profiles along $x$. The increase of the amplitude of each harmonic at the maxima of their respective $y$-profiles (figure 1*b*), shows that they not only demonstrate no growth in the region of spike formation, but that on

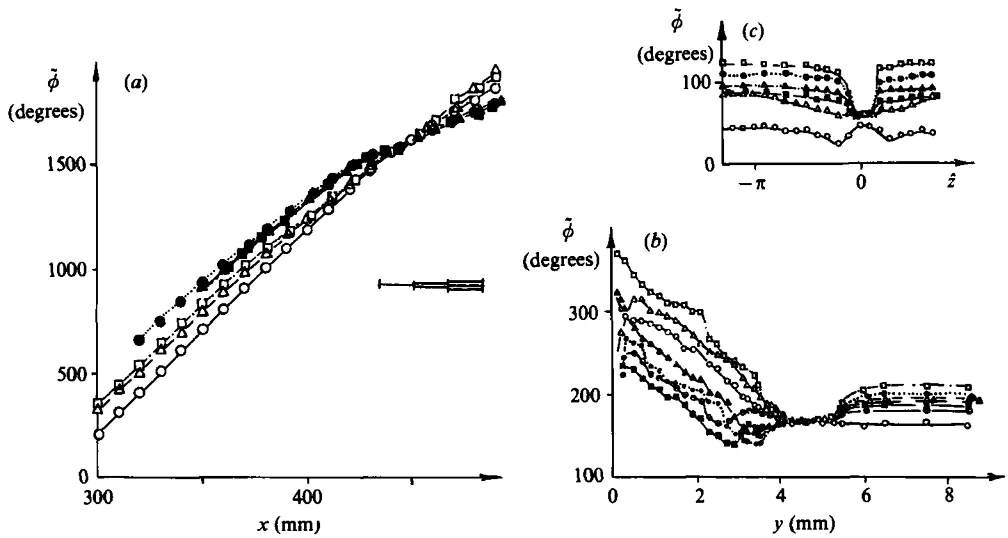


FIGURE 2. Synchronization of harmonic phases in the places where a spike appears (from Kachanov *et al.* 1984). Phases $\tilde{\phi}$ are measured in degrees of fundamental period: (*a*) peak, $y = 4.5$ mm; (*b*) peak, $x = 450$ mm; (*c*) $x = 430$ mm, $y = 4.5$ mm.

the contrary they decrease. It was found that the spike formation is connected not with a jump in the disturbance amplitude, but with the synchronization of the harmonic phases in definite regions of space. Figure 2 shows this synchronization observed in local distributions along $x$, $y$, $z$, in the regions where the spikes appear in the oscilloscope traces.

The frequency–wavelength Fourier analysis of the data conducted by Kachanov *et al.* (1984) shows that, while the spikes are forming, the amplitudes of three-dimensional harmonics of the frequency–wavelength spectrum, inclined at a definite angle to the flow, gradually amplify. The increments in their amplification $\kappa$ are 1 or 2 orders of magnitude greater than those for the plane wave of frequency $\omega_1$, as given in figure 3. Simultaneously, in the phase frequency–wavelength spectra obtained at the $y$-coordinate of spike formation, synchronization of the phases of amplified frequency–wavelength harmonics is observed (figure 4). These are all reasons to suppose that the process of spike formation represents a resonant amplification of 'subharmonics' of the type $(n\omega_1, \alpha_n, \pm\beta_n)$ under the effect of plane waves $(2n\omega_1, 2\alpha_n, 0)$ according to the Csaik–Nayfeh–Bozatli mechanism studied for four waves by Nayfeh & Bozatli (1979*a*). The selectivity of the parametric resonances to the phase of the generated waves may naturally explain the phenomenon of phase synchronization of a frequency–wavelength spectrum and, as a result, may explain the appearance of spikes in oscilloscope traces.

The aim of the second part of the present work is to analyse critically the existing experimental data on disturbance development and flow breakdown from the viewpoint of the proposed lower wave-resonance concept and in particular to compare this new approach with the concept of local high-frequency secondary instability.

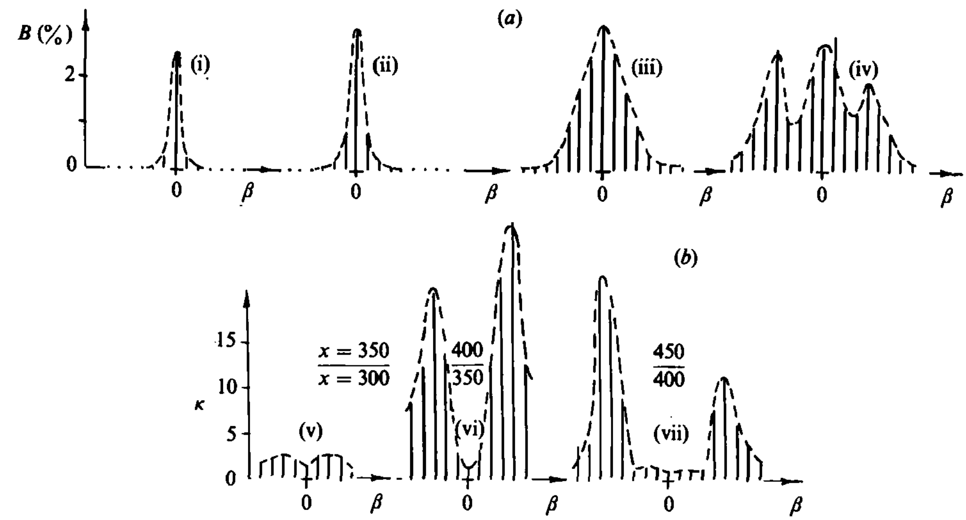


FIGURE 3. (a) Frequency–wavelength spectra for frequency $\omega_1$ ($y = 1$ mm) at: (i), $x = 300$; (ii), 350; (iii), 400; (iv) 450 mm, and (b) their ratios: (v), (ii)/(i); (vi), (iii)/(ii); (vii), (iv)/(iii) (from Kachanov *et al.* 1984).

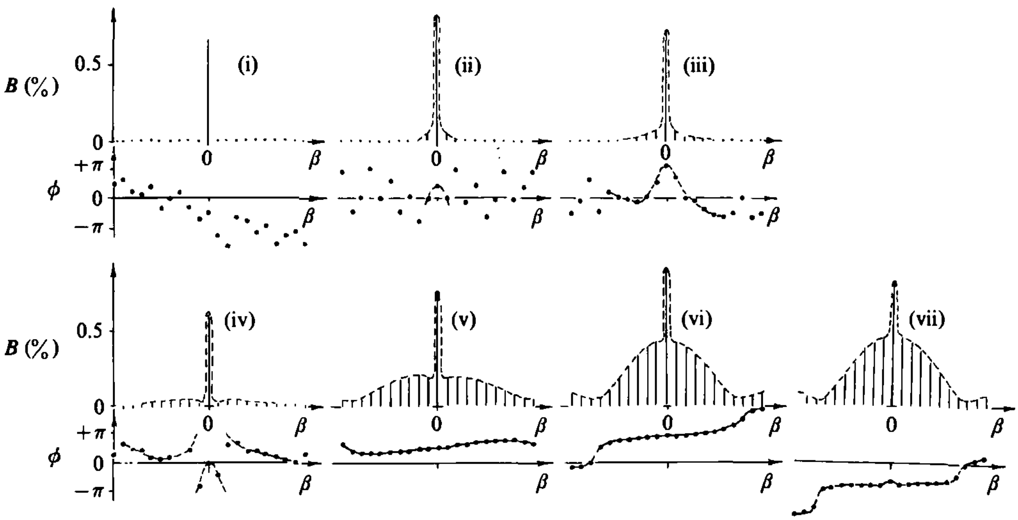


FIGURE 4. Frequency–wavelength amplitude and phase spectra for frequency $\omega_1$ ($y = 4.5$ mm): (i)–(vii), $x = 300$, 350, 400, 420, 430, 440, 450 mm respectively (from Kachanov *et al.* 1984).

## 3. Wave-resonance concept

### 3.1. *On the correspondence between wave packets and their spectra*

The object of this section is to demonstrate the correspondence that exists between characteristics of wave packets (as fluctuations in time) and properties of their frequency and frequency–wavelength spectra. These properties are well known, but they are very important for the subsequent discussion.

Time-periodic flashes having an identical form are an example of simple packets (figure 5*a*, curve i). Their frequency spectrum consists of the high harmonics $\omega_n = n\omega_1$ ($n = 2, 3, 4\ldots$) of the fundamental frequency $\omega_1$, which are shown in figure 5*b* (curve

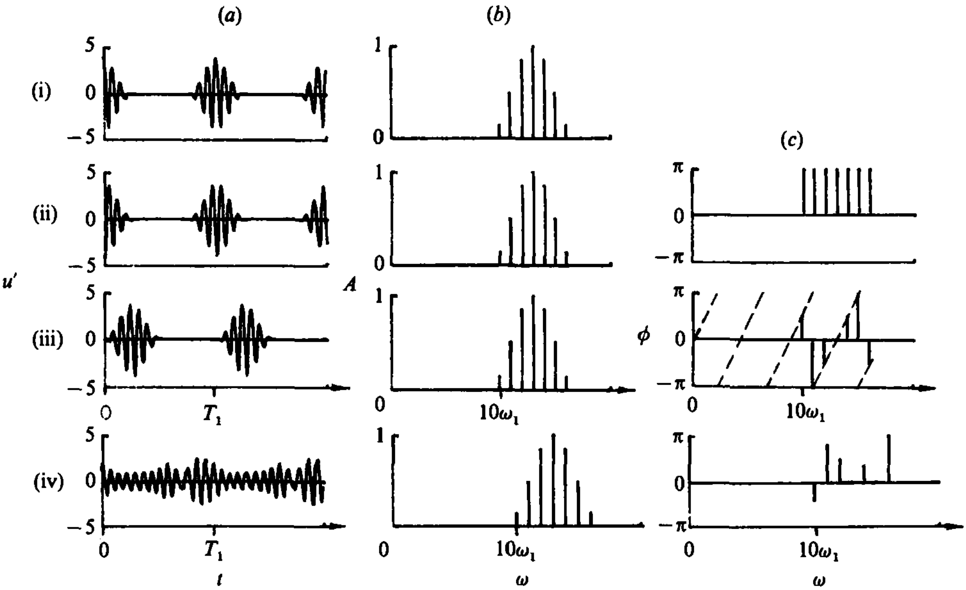


FIGURE 5. (*a*) Simple wave packets and (*b*) their amplitude, and (*c*) phase spectra.

i). The phases $\theta_n$ of the packets in curve (i) are equal to zero. (Here and subsequently the waves $A_n \cos(\theta_n - n\omega_1 t)$ are considered.) The flash takes place at $t = 0$ because there all harmonics are superposed in phase. Then the phases are disordered and the harmonics interfere. After a period $T_1 = 2\pi/\omega_1$ the phases coincide again with the same value as at $t = 0$. Therefore the flash appears again near $t = T_1$.

The displacement of the phases of all the harmonics by the same value $\Delta\theta$ leads to synchronization (taking place in each period) not of the crests of the waves (as in the case $\theta_n = 0$), but of the characteristic points with the phases $\chi = \Delta\theta$. Curves (ii) in figure 5 correspond to the displacement $\Delta\theta = \pi$, leading to the periodical synchronization of the wave valleys, but not of crests (see curves (i) in figure 5). In other words, the shift $\Delta\theta$ leads to the displacement of the 'phase' of quasi-sinusoidal oscillations within the packet by an angle $\approx \Delta\theta$.

When the phases of the harmonics change by the angles $\Delta\theta_n = n\chi$, a shift of the moment of phase synchronization (i.e. of the packet centre) $\Delta t = \chi/\omega_1$ takes place. The phases of characteristic synchronized points also change by a value $\chi$. An example of such phase shift, when $\chi = \frac{1}{2}\pi$, is shown in figure 5 (curves iii).

Phase spectra for curves (ii) and (iii) are shown in figure 5(*c*). The phases of the harmonics play a prime role both in the appearance of wave packets and in their properties. Curves (iv) in figure 5(*a–c*) demonstrate what happens to the packets when the harmonic phases are disordered (in this case they have random values). The packets are shown to disappear.

What will happen to the packets if the frequencies of spectra (like those in figure 5*b*) are shifted by some value $\Delta\omega$? An example of such packets is shown in figure 6(*a*). Their spectrum (figure 6*b*) coincides in form with the spectra in figure 5(*b*). The initial phases of all the harmonics are equal to zero (the same as for curves (i) in figure 5). However the frequency shift $\Delta\omega = 0.25\omega_1$ leads to a variation in the shape of the flashes with time, though they are observed in each period $T_1$ as before. It is clear that after $n$ periods the characteristic points with phases $\chi_n = n\,\Delta\omega T_1 = n(\Delta\omega/\omega_1)\,2\pi$

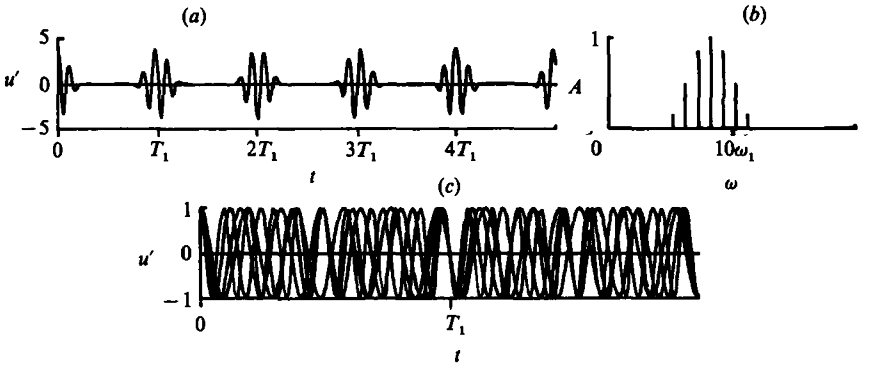


FIGURE 6. (*a*) Packets with moving phase, (*b*) their spectrum and (*c*) demonstration of phase synchronization.

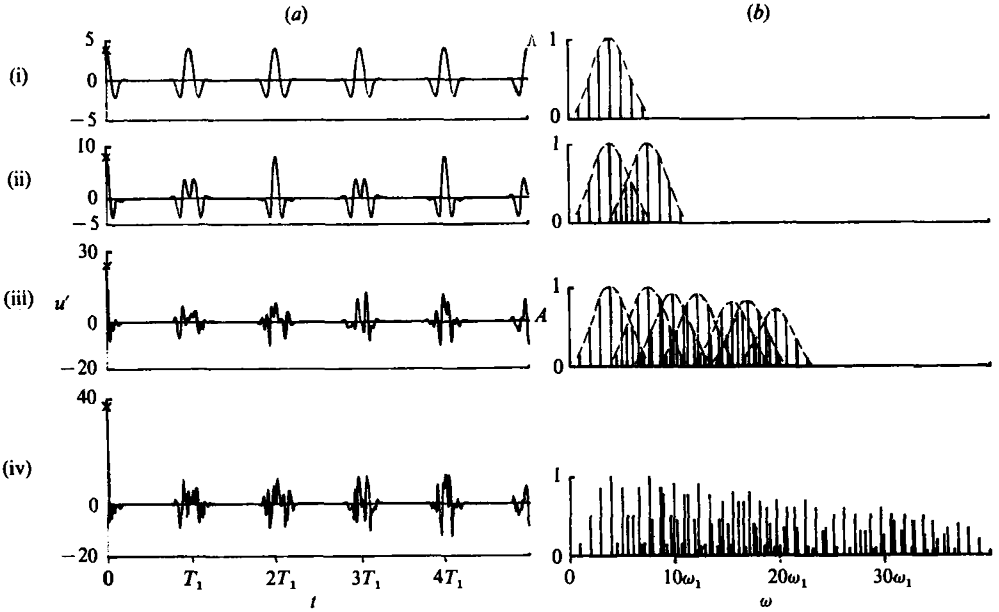


FIGURE 7. Spectral model of the intermittency phenomenon.

will be synchronized. Figure 6(*c*) demonstrates this fact graphically, showing 5th, 7th, 8th and 10th harmonics shifted by the value $+0.25\omega_1$.

Finally, it is interesting to consider the case when a spectrum consists of a sum of the spectra (like those in figure 6*b*) having different $\Delta\omega$ and $\omega_m$, but the same $\omega_1$. This is shown in figure 7 (*b*), with the corresponding oscilloscope traces shown in figure 7 (*a*). The initial phases $\theta_n$ for all the harmonics in figure 7 are equal to zero, leading to synchronization of crests at $t = 0$ (marked by a cross). When the number of groups of harmonics, having random frequency shifts $\Delta\omega_k$, growing mean frequency $\omega_m$ and slowly decreasing amplitudes, is increased (see curves i–iv), the flashes acquire a quite complex character. The oscillations 'forget' their zero initial phases very quickly (at $t > 0$) but they do not 'forget' the fundamental period $T_1$.

Curves (iv) in figure 7 (*a*) can be regarded as a spectral model of the intermittency studied by Kachanov, Koslov & Levchenko (1978*a*) for the boundary layer and by Miksad (1973) for free-shear flow.

### 3.2. *Wave-resonance concept for* K-*breakdown*

As noted above, I contained a suggested explanation of the main causes of the difference between the classical K-regime of laminar-boundary-layer breakdown (see Klebanoff *et al.* 1962) and the new N-regime described in I, based on the analysis of experimental and theoretical data. This difference was explained by the predominance in the K-regime (when the initial amplitudes are higher) of the parametric resonances between waves

$$(2n\omega_1, \alpha_{2n}, 0), \tag{1}$$

$$(n\omega_1, \alpha_n, \pm\beta_n), \tag{2}$$

which amplifies, in particular, a determinate set of priming waves (2). In contrast the N-regime displays subharmonic resonance between waves

$$(\omega_1, \alpha_1, 0), \tag{3}$$

$$(\tfrac{1}{2}\omega_1, \alpha_{\frac{1}{2}}, \pm\beta_{\frac{1}{2}}), \tag{4}$$

which amplifies random priming waves near the frequency $\frac{1}{2}\omega_1$. The overriding importance of the resonances of waves (1) and (2) in the K-regime is conditioned by the fast generation of plane high harmonics

$$(m\omega_1, \alpha_m, 0) \tag{5}$$

of the fundamental wave as a result of harmonic resonance. Just as the existence and the dominating role of the resonance of waves (3) and (4) in the N-regime of the breakdown were discovered experimentally and substantiated in II, so the existence of harmonic resonances for the $m$-plane waves and the existence of parametric resonances (1) and (2) (when $m, n > 2$) are postulated here as a generalization of the two-wave harmonic resonance studied by Nayfeh & Bozatli (1979*b*), and of the four-wave parametric resonance of (1), (2) and (5) (when $m, n = 1$) proposed by Craik (1971) and studied by Nayfeh & Bozatli (1979*a*). This generalization may be admitted in view of two main circumstances. First, the presence of the strict hierarchy of high harmonic amplitudes that was found for the K-regime by Kachanov *et al.* (1984) in the stage up to the spike appearance, and a monotonic decrease of their amplitudes according to the law of geometric progression when the $m$-number grows (see figure 8*a*), testify to the validity of the weakly nonlinear approach and to the efficiency of the spectral representation in this stage of the development. Secondly, the presence of a strong resonant coupling of waves (1) and (2) gives us every reason to think that the development of the resonant sets (in a simple case, of triads) proceeds independently in the first approximation.

Nayfeh & Bozatli (1979*a*) noted that in the case of the four-wave resonance for waves (1), (2) and (5) at $n$, $m = 1$ there 'take place two interaction processes at the same time. The first one is the interaction between a two-dimensional fundamental wave and its second harmonic. As shown by Nayfeh & Bozatli (1979*b*), this is a strong destabilizing mechanism for the second harmonic. In the second part of the interaction, the second harmonic interacts with its two three-dimensional subharmonic waves of order one-half and produced large increases in the amplitudes of the three-dimensional waves'.

Very important properties of the parametric amplification of the instability waves were found in II, namely (i) an exceptionally large spectral width of the resonance, (ii) the property of symmetrizing a spectrum of the amplified disturbances relative to the subharmonic frequency, and (iii) a very wide zone of parametric amplification (without the reverse influence of the subharmonic on the fundamental wave) valid even for subharmonic amplitudes exceeding the amplitudes of the fundamental wave. These properties lead to a number of important conclusions when the generalization for the case of the $m$-resonances is carried out.

First, besides the harmonics with frequencies $n\omega_1$ (i.e. subharmonics for waves $2n\omega_1$), because of the observed large width of the resonance ($\Delta\omega \sim \frac{1}{2}\omega_1$), a number of neighbouring modes

$$(k\omega_1, \alpha_{kl}, \pm\beta_l), \quad \tfrac{1}{2}n \lesssim k \lesssim \tfrac{3}{2}n \tag{6}$$

will also lie within the resonance zone.

Consequently, the three-dimensional waves can be generated parametrically not only by harmonics of type (1) but also by odd harmonics

$$((2n-1)\,\omega_1, \alpha_{2n-1}, 0), \tag{7}$$

which do not have their own priming subharmonic of frequencies $(2n-1)\frac{1}{2}\omega_1$ with large amplitudes but which can amplify waves of type (6).

In view of property (ii), parametric amplification will lead to a fast, predominant amplification of the high-frequency harmonics $(n+l)\,\omega_1$ ($l \lesssim \frac{1}{2}n$), which will be symmetrized with the low-frequency harmonics $(n-l)\,\omega_1$ having relatively large amplitudes. In fact, the mechanism of symmetrization (ii) (discussed in detail in II) means that the harmonics $(n-l)\,\omega_1$ play the role of priming waves for the amplification of the harmonics $(n+l)\,\omega_1$. The property (iii) means that, in the present case, such an amplification may drive the amplitudes of the harmonics (6) up to the amplitude of the plane forcing wave like (1) or (7). Moreover, the superposition of these waves (owing to the overlapping of the resonant zones for different $n$), synchronized with respect to the phase, may give spatially localized excesses of the amplitudes of resonantly amplified three-dimensional modes (6) over the amplitudes of plane waves generated by harmonic resonance.

Note two more circumstances. An essential difference between harmonic resonances and the parametric subharmonic ones is that the former themselves generate the oscillations with high harmonic frequencies; but initial, priming, waves in the spectral region of the resonance are necessary for realizing the parametric amplification. Without priming this latter amplification will not always be observable (see, for example, II).

Another difference of this resonance (or near-resonance) interaction arises from the nature of the amplified modes. The parametric resonance amplifies free (eigen) modes of the boundary layer of type (2), (4) and (6) or the modes close to them (because of nonlinear distortion). Disturbances of type (1), (5) and (7), which are amplified by harmonic (and combination) resonance, really consist of the superposition of bound (pure nonlinear) oscillations and free (eigen) ones, generated by nonlinearity.

Figuratively speaking, the wave-resonance concept for the case of K-breakdown may be formulated as follows. The harmonic resonances pave the way through a sparsely inhabited spectrum for the disturbances, generating the plane high harmonics (that drive the parametric resonances), with smaller three-dimensional disturbances (being the priming waves for the beginning of parametric amplification). The parametric resonances amplifying three-dimensional harmonics in a wide region of low frequencies proceed in this way.

The qualitative scheme proposed here requires (i) an estimate of its experimental correctness and (ii) carrying out corresponding theoretical estimations and calculations. The next sections of this work concern the first of these two purposes. An analysis of wave kinematics will be performed, with the aim of clarifying the properties (especially the phase properties) expected of disturbances amplified by resonances and comparing them with the phenomena actually observed.

## 4. Phase properties of resonantly amplified waves: comparison with experiment

### 4.1. *Harmonic resonance of plane waves*

Usually in theoretical studies (in particular, in the work of Nayfeh & Bozatli 1979*a*, *b* where an analysis of such resonance is carried out) special attention is paid to the amplitudes of resonantly amplified waves and also to their distributions in space. However, the phase properties are very important for understanding the nature of the phenomena observable in the experiments of K-breakdown (see the work by Klebanoff *et al.* 1962 and Kachanov *et al.* 1984).

The nonlinear harmonic resonance of plane waves leads to the generation of two-dimensional high harmonics of the fundamental wave of type (5), which in the plane-parallel approach can be represented as

$$u_n^{(2)}(x,y,t) = A_n(x,y)\exp\left[\mathrm{i}(\alpha_n x - \omega_n t + \phi_n)\right]. \tag{8}$$

Here $A_n$ is the complex wave amplitude, $\alpha_n$ is the wavenumber, $\omega_n$ is the frequency, and $\phi_n$ is the initial phase. We also designate $\alpha_n x + \phi_n = \theta_n(x)$.

As is known, satisfaction of the phase synchronism conditions (see Nayfeh & Bozatli 1979*b*) is necessary for the manifestation of harmonic resonance. These conditions for the case of $n$ waves are

$$\begin{cases} \omega_n = n\omega_1, \\ \theta_n = n\theta_1. \end{cases} \tag{9}$$

The second of these conditions is not exactly realized in practice because of a small degree of dispersion. But for the phases of real, resonantly amplified waves this detuning can be compensated partially or completely owing to the phenomenon of phase capture, which was certainly observed in II for the case of parametric resonance.

The condition (9) means (see §3.1) synchronization of the phases of all harmonics at the time moment $\Delta t = \theta_1/\omega_1$, at those characteristic points with phase $\chi = \theta_1$ (see figure 5, curves iii). The difference from the situation considered in §3.1 consists only in the fact that the wave packet amplified by the harmonic resonance begins with the harmonic $\omega_1$ and contains all higher harmonics decreasing in amplitude. In figure 8(*a*), obtained from Kachanov *et al.* (1984), it can be seen that in the K-regime the amplitudes of the high harmonics, as determined by their maxima in the $y$-profile, decrease almost exactly in accordance with a geometric progression with a factor $q \approx 0.32$ at the point $x = 350$ mm (long before the spike formation), and they decrease exactly in this way with the factor $q = 0.48$ at $x = 400$ mm.

A model of such an amplitude spectrum with $q = 0.32$ is shown in figure 8(*b*). The traces represented in figure 8(*c*) (curve i) correspond to this spectrum at $\theta_1 = 0$. (It is clear that it is possible to make the phase $\theta_1 = 0$ by an appropriate shift of the time axis.) The usual normalization of phase profiles employed in the theoretical work, that $\theta \to 0$ at $y \to \infty$, means that such a synchronization of the wave crests should be observable near the boundary-layer top, i.e. in the region that is higher than the phase jumps observable for all the harmonics in the experiment of Kachanov

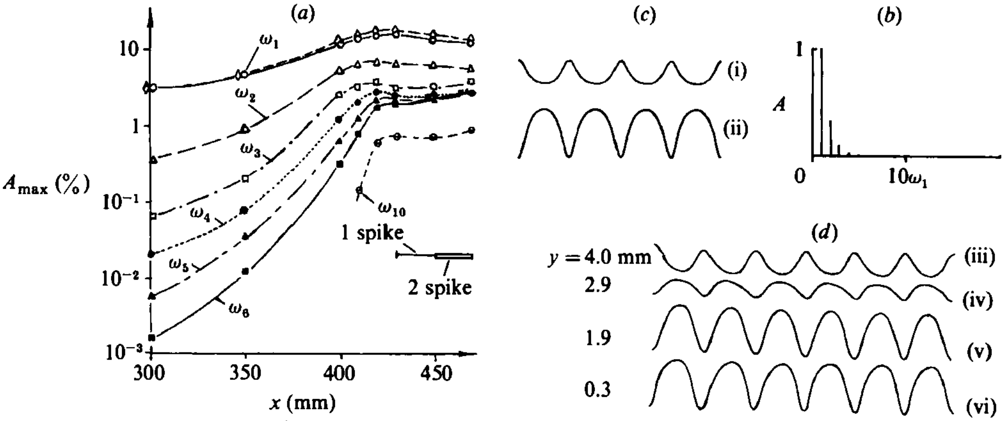


FIGURE 8. (*a*) Amplification of maximal amplitudes of frequency harmonics during spike formation in the peak position; (*d*) oscilloscope traces at $x = 350$ mm, and (*c*, *b*) their simulation (based on the data of Kachanov *et al.* 1984).

*et al.* (1984) and in the experiment performed by Gilyov, Kachanov & Kozlov (1981). This synchronization is actually observed in Kachanov *et al.* (1984). The experimental curve (iii) in figure 8 (*d*) taken from Kachanov *et al.* (1984) corresponds to the region above the jumps ($x = 350$ mm, $y = 4$ mm); this is in good qualitative agreement with curve (i) in figure 8 (*c*).

It is clear that, because of the dependence of the wave phases on $y$, other oscilloscope traces should be observed on moving towards the wall. It was demonstrated in §3.1 that the phase shift of all the harmonics by the same angle (i.e. $\Delta\theta_n \equiv \Delta\theta$) leads to displacement of the phase angle of the synchronized characteristic points. Curve (i) in figture 8 (*c*), as noted, corresponds to the synchronization of the crests, i.e. of the characteristic points having the phase $\chi = 0$. As a result, the crests of the summed wave are somewhat sharpened. After shifting the probe towards the wall through the phase jumps of all the harmonics that were observed in the experiments of Kachanov *et al.* (1984) (see also figure 9*a*), characteristic points having phase $\chi = \pi$ (i.e. the wave valleys) will be synchronized. For the model spectrum in figure 8 (*b*) such a situation corresponds to the oscilloscope trace (ii) in figure 8 (*c*). Indeed, just such a change in the form of the oscilloscope traces is observed below the jumps in the experiment of Kachanov *et al.* (1984) (curves (iv)–(vi) in figure 8*d*). The synchronization of the harmonic valleys leads to sharpening of the valleys of the summed oscillations.

### 4.2. *Parametric amplification of three-dimensional harmonics*

In accordance with the wave-resonance concept presented in §3.2, the harmonic resonance amplifies two-dimensional harmonics $\omega_m$ of higher and higher frequency and creates the conditions for parametric resonant amplification of three-dimensional waves. As noted above, three-dimensional priming disturbances in the range of frequencies from $\sim(n-l)\,\omega_1$ to $\sim(n+l)\,\omega_1$ are necessary for this amplification. Here $l \sim \frac{1}{2}n$ and $m = 2n$ or $2n+1$. The most intensive priming disturbances in this frequency range in the K-regime coincide with the frequencies of the harmonics $k\omega_1(n-l \lesssim k \lesssim n+l)$. A combination of the nonlinear interaction of plane harmonics with the non-uniformity of the mean flow in the $z$-direction can create three-dimensional priming waves for the subsequent parametric resonances. In the con-

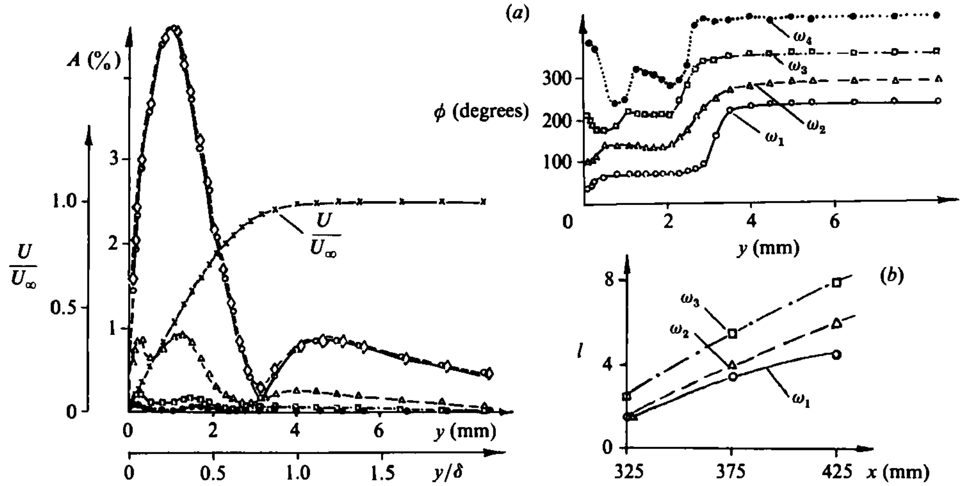


FIGURE 9. (*a*) Amplitude, phase and mean velocity profiles in the peak position at $x = 350$ mm, and (*b*) growth of wavenumber of most amplified frequency–wavelength harmonics along $x$ for different frequencies.

trolled conditions in the experiments of Klebanoff *et al.* (1962) and in Kachanov *et al.* (1984), the wavelength spectra (obtained as a result of Fourier transformation in the $z$-direction) consist of harmonics $l\beta_1 (l = 1, 2, \ldots)$ of the wavenumber $\beta_1 = 2\pi/\lambda_{1z}$, where $\lambda_{1z}$ is the chosen interval of the spacers (see figures 3 and 4).

The well-known property of parametric resonance of amplifying only those disturbances having definite values of phase $\phi_r$ or $\phi_r + \pi$ was shown in the experiments of II; these experiments also determined that the value $\phi_r$ is equal to $\pi$ in the region below the phase jumps in the $y$-profiles of disturbances. It corresponds to $\phi_r = 0$ in the region above the phase jumps. What is the actual phase structure of the frequency–wavelength priming harmonics, generated by a combination interaction in the framework of the wave-resonance concept, and how does it correlate with the resonant values of the phases that are necessary for the realization of parametric amplification and for the spike formation in the K-regime?

In the first section ($x = 300$ mm) in the experiments of Kachanov *et al.* (1984) we have an initial three-dimensional stationary mode (actually a spanwise-periodic modulation of the mean flow) of the type $(0, \beta_1)$. The initial phase of this mode is equal to zero when the origin of the $z$-axis is placed in the peak position. Nonlinear combination interaction of this mode with the plane fundamental wave and with its higher harmonics gives modes of the type

$$(n\omega_1, \pm l\beta_1). \tag{10}$$

It is not difficult to show that all the harmonics of this type must have zero phase at the point $z = 0$.

Thus, the priming disturbances for the parametric resonances in the K-regime are found to have at the peak position ($z = 0$) the most favourable (the phase $\pi$ is favourable too) and identical resonant initial phases. This very important fact determines the occurrence of the K-breakdown.

The determinism of the priming disturbances and their coherence with the forcing wave are the essential difference between the parametric resonances in the K-regime at the stage of spike appearance and those in the N-breakdown. Another important difference consists in the presence in the K-regime of a set of forcing waves having priming disturbances of the same frequency. It was shown experimentally in II that the wavenumber $\beta$ for the resonantly amplified oscillations is increased when the frequency of the forcing wave is increased, the travelling angles of the amplified waves being roughly unchanged $\hat{\theta}_{\frac{1}{2}} \approx \pm 60°$. This means that, because of the large spectral width of the resonances (observed in II), each of the frequency harmonics, especially those with large $k$, will participate simultaneously in a number (may be in tens) of resonances having different frequencies of forcing waves. Therefore, at the fixed frequency $\omega_k = k\omega_1$ of this harmonic, the different resonances will amplify three-dimensional waves with different wavenumbers $\beta_l = l\beta_1$. The higher the frequencies of the forcing waves $\omega_m$ the larger are their wavenumbers $\alpha_m$ and the larger the wavenumbers $\beta_l$ of the amplified three-dimensional waves. This means that more and more high harmonics $\beta_l$ will be amplified downstream because of the growth of amplitude of plane high-frequency harmonics (see §4.1). This is observed in the experiment of Kachanov *et al.* (1984). The graph shown in figure 9(*b*) is drawn on the basis of the table from that work. The dependence of $l$, the number of the most amplified three-dimensional harmonics $\beta_l = l\beta_1$ for the frequencies $\omega_k = \omega_1, \omega_2, \omega_3$, on the downstream distance is shown. The number $l$ grows downstream and grows with increasing $\omega_k$.

The amplitudes of the amplified three-dimensional waves will quickly grow, especially for $k \geqslant \frac{1}{2}m$, because of the effect of symmetrization of the form of the spectrum relative to the subharmonic frequencies (see §3.2). This increases the amplitudes of harmonics $\frac{1}{2}m + l$ up to the amplitudes of harmonics $\frac{1}{2}m - l$ having the same wavenumbers $\beta_l$. As was noted in §3.2, the parametric, linear character of the resonance (i.e. the absence of the reverse influence of the amplified waves on the forcing wave) persists up to amplitudes $A_{kl} \sim A_{mo}$. It can be seen from the experiments of Kachanov *et al.* (1984) that the amplitudes of three-dimensional frequency–wavelength harmonics $A_{1l}$ grow to values $\sim 2\%$, when the amplitude of the plane forcing wave $A_{2o}$ is only $\sim 1\%$. Taking into account the generation of $A_{ol}$ by two different resonances and the dependence of $A_{2o}$ on the $y$-coordinate, it can be concluded that in this respect the experimental results of Kachanov *et al.* (1984) are also in good agreement with the wave-resonance concept.

Let us consider now the results of such resonant parametric amplification of priming waves of type (10). First we add up the waves (+) and (−) of type (6) since, by reason of symmetry relative to the $x$-axis, such waves are experimentally observed to occur in pairs. (Zelman & Maslennikova (1982) found that in asymmetric cases the resonance symmetrized the subharmonic (+) and (−) in the same manner as did the frequency spectrum in the experiments of II.) The superposition of these waves gives the usual picture of a standing wave. Their initial phase, as before, is equal to zero near the peak position ($z = 0$) and also near the points $z = \pm 2k\pi/\beta_l$.

The resonant parametric interaction will lead, at $z = 0$ (i.e. in peak locations) to amplification and to symmetrization of the amplitudes of pairs of priming waves with frequencies $\omega_k = \omega_s \pm \Delta\omega_s$, where $\omega_s$ is the subharmonic frequency (see II). The superposition of such a pair of resonantly amplified harmonics creates a beating of amplitudes in time according to the law $|\cos(\Delta\omega_s t)|$, and phase jumps in the moments when the amplitudes are equal to zero.

An example of such superposition of three different resonantly amplified pairs of disturbances is modelled in figure 10 (curves iii–v). Curve (vi) corresponds to the amplified coherent subharmonic with frequency $\omega_s = 10\omega_1$. Curves (iii)–(v) correspond to the superposition of resonantly amplified and symmetrized pairs of waves

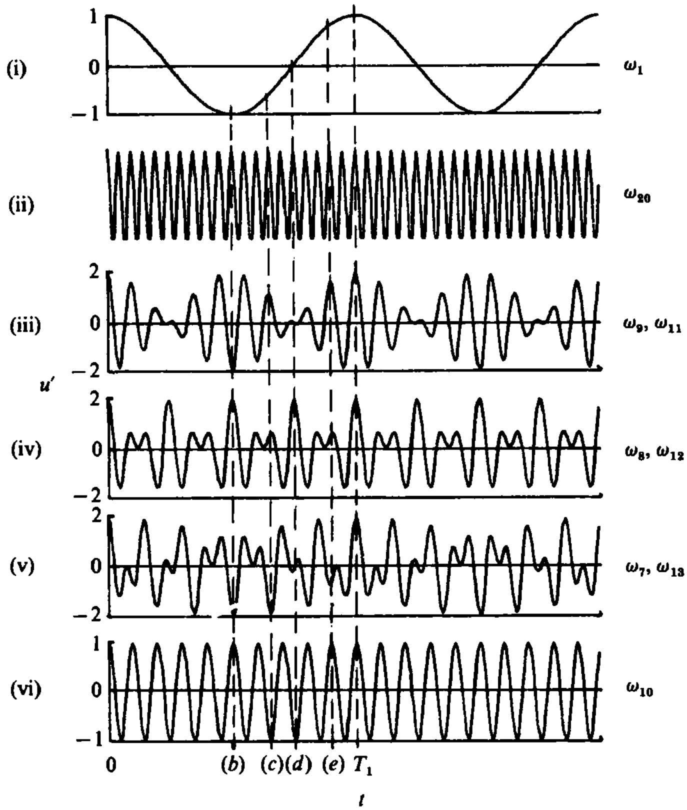


FIGURE 10. Simulation of parametric amplification of some high-frequency harmonics of the fundamental wave.

$\omega_9, \omega_{11}$; $\omega_8, \omega_{12}$ and $\omega_7, \omega_{13}$. Curve (ii) is the forcing wave $\omega_{20} = 20\omega_1$ and curve (i) is the fundamental wave $\omega_1$.

Figure 10 demonstrates that the resonantly amplified subharmonics and quasi-subharmonics are locally in phase or in antiphase with the forcing wave (compare with figure 6 in II). Because the detunings $\Delta\omega_s$ are divisible by $\omega_1$, the synchronization of the initial phases with $\chi = 0$ (of the wave crests), which is favourable for the resonance, is reproduced periodically at the times $kT_1$ for all $\Delta\omega_s$. For other time moments, however, the resonant amplification of individual frequency pairs of harmonics is still present (moments (*b–e*) in figure 10, for example), but the phases of the characteristic synchronized points are different for the different pairs (valleys or crests). The analogous situation is observed for other forcing harmonics $\omega_m$.

The superposition of quasi-subharmonic waves such as those shown in figure 10 (curves iii–vi) gives, for different $\omega_m$, the curves shown in figure 11 (*a*) and labelled (ii), (iii) and (iv) for $m = 20$, 18 and 19 respectively. The amplitude spectra corresponding to these curves are shown in figure 11 (*b*). These spectra reproduce qualitatively a wave packet being amplified by each of three resonances with forcing waves $\omega_{20}$, $\omega_{18}$ and $\omega_{19}$, respectively (see spectra in figures 4 and 14*a* in II). The characteristic amplitude of these packets grows as $\omega_m$ and $\omega_s$ decrease, since the amplitudes of plane forcing waves $\omega_m$ and priming waves with frequencies $\omega_s \pm \Delta\omega_s$ then grow too.

Superposition of a set of such packets, together with its spectrum, is shown in figure

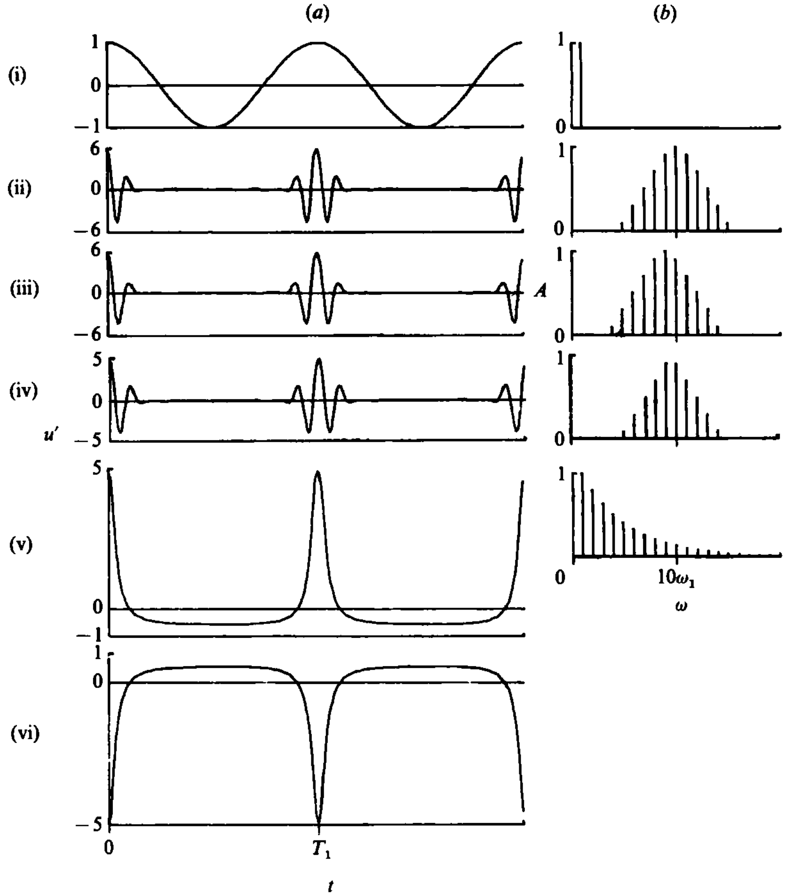


FIGURE 11. Formation of spikes by parametric resonances.

11 (*a*, *b*) (curves v). This amplitude spectrum models one actually observed in the experiments of Kachanov *et al.* (1984) at $x = 420$ mm, where the harmonic amplitudes decrease according to the geometric progression with the factor $q = 0.8$ (see figure 8*a*), but the chosen phases of harmonics for the trace correspond to the wave-resonance concept. The point of phase synchronization for all the harmonics is displaced by $\pi$ after crossing the phase jumps of harmonics as $y$ diminishes (see §5.1 and figure 12*a* below). Therefore, in the region lying below the jumps, the oscilloscope traces have the form shown in curve (vi) of figure 11.

So, as seen, the nonlinear development of the primary wave in a spanwise-modulated mean flow must lead, from the wave-resonance point of view, to the appearance of oscilloscope traces having spikes which are typical for the K-regime of breakdown.

Besides the experimental facts shown to be well explained in the framework of the proposed wave-resonance concept, a great number of other features observed in the K-regime as well as in the N-regime of breakdown can be qualitatively explained and understood from this viewpoint. This explanation is concerned with different distributions in the space $(x, y, z, t)$, a number of which will be demonstrated in the next sections.

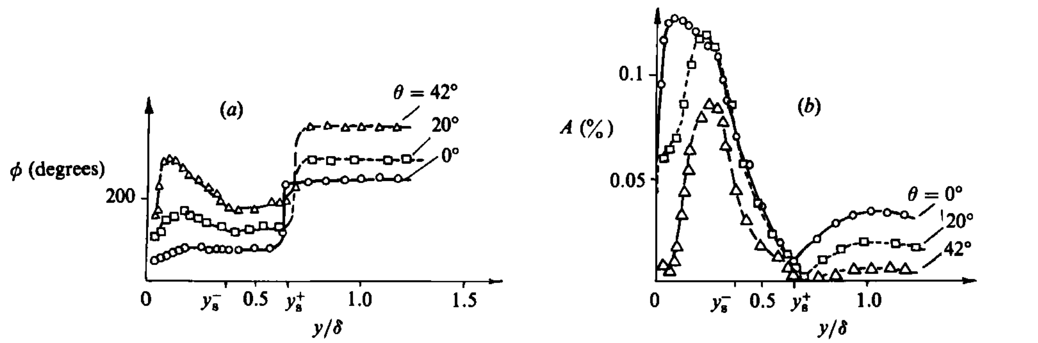


FIGURE 12. Dependence of (*a*) phases and (*b*) amplitudes of eigenfunctions of three-dimensional waves in a flat-plate boundary layer upon propagation angle $\theta$, measured by Gilyov *et al.* (1981). $Re^* = 1220$, $F = 94 \times 10^{-6}$.

## 5. Properties of $t$, $y$, $z$ distributions: explanation of experimental data

### 5.1. *Oscilloscope traces and y-profiles*

Why do we usually observe spikes only in the external part of the boundary layer (below the jumps of the harmonics' phases) and not at other distances from the wall?

From the general properties of the parametric resonances and from a comparison of the experimental results of II and those by Gilyov *et al.* (1981) the conclusion follows that the form of the $y$-profiles for frequency–wavelength harmonic amplitudes and phases, which are resonantly amplified, is close to the profiles of the linear three-dimensional eigenwaves. These profiles (eigenfunctions measured by Gilyov *et al.*) are shown in figure 12. Since in the K-regime parametric resonance amplifies a wide spectrum of three-dimensional waves having different $\beta_l$ for each of the frequencies, the cause of the spikes vanishing in the oscilloscope traces at $y < y_s^-$ becomes clear. This phenomenon takes place as a result of the detuning of the frequency harmonic phases because the higher is the harmonic frequency, the larger are typical values of $\beta_l$ in its wave spectrum (see figure 9*b*), and the more its phase starts to change towards the wall (see figure 12*a*). The lack of coordination of the phases of different frequency harmonics destroys the spikes in the oscilloscope traces.

The cause of the 'absence' of the spike in oscilloscope traces in the region $y > y_s^+$ is connected with the fast decrease of amplitudes of all the frequency harmonics above the phase-jump zone. In their turn the small oscillation amplitudes, observed higher than the phase jumps, are connected with the smallness of the eigenfunction 'tails' for three-dimensional waves (see figure 12*b*). Their amplitude decreases very quickly with the growth of $\beta$. Nevertheless, the harmonic phases remain in synchronism at $y > y_s^+$, the wave crests being synchronized. This circumstance should generate a spike that is orientated upwards and has a small amplitude. This 'small spike' is actually observed in the experiments of Kachanov *et al.* (1984) (as well as in the work by Klebanoff *et al.* 1962) at the peak position when $y > y_s^+$.

The properties of the phase profiles of three-dimensional waves also provide a possible explanation, in the framework of the wave-resonance concept, of the appearance of the second spike (just in the region $y < y_s^-$) and of some of its properties.

The wave-resonance concept also permits an explanation of the form of the frequency-harmonic-amplitude profiles at the stage before the spike appears, where the local secondary instability of the flow was studied by Nishioka, Asai & Iida (1980). The corresponding profiles taken from Kachanov *et al.* (1984) are shown in

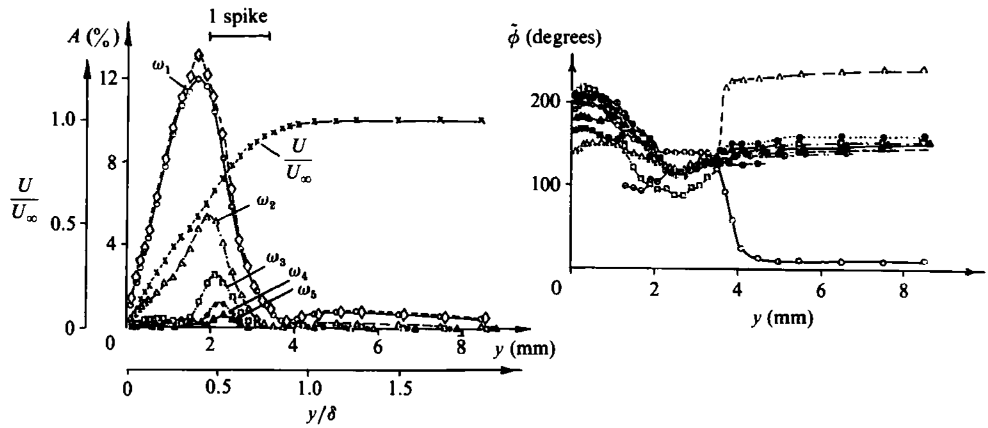


FIGURE 13. Amplitude, phase and mean-velocity profiles at the peak position at $x = 400$ mm (from Kachanov *et al.* 1984).

figure 13 for the peak position and at $x = 400$ mm (in the terminology of Nishioka *et al.* (1980) it corresponds to the 11.5% stage). It is seen that the amplitudes of all the frequency harmonics at first grow at $y \lesssim y_s^+$ but then they quickly decrease in turn in the region $y \sim y_s^-$ to very small values, beginning with the highest frequency harmonic. In contrast, the first and second harmonics change rather slowly. From the wave-resonance point of view this peculiarity of the amplitude profiles is explained by the interference of the frequency–wavelength harmonics having different $\beta_l$ but the same $\omega_k$. A lack of phase coordination occurs because of the differences in the phase profiles observed near the wall for the different $\beta$ (see figure 12*a*). The harmonics with higher frequencies start to decrease farther from the wall because there the wavelength spectrum is richer and further displaced towards high $\beta$ (see §4.2).

The properties of the $y$-profiles observed in the valley position – in particular their double-hump form for the fundamental wave – can also be explained in the same way (and it is close to the explanation given by Craik 1980). Briefly, its essence is that waves $(\omega_1, 0)$ and $(\omega_1, \pm\beta_1)$, which have the same phases in the peak position, are in antiphase in the valley position (see §4.2) and their amplitudes are of the same order (see Kachanov *et al.* 1984). The subtraction of these waves, with their individual amplitude $y$-profiles (measured by Gilyov *et al.* 1981 see figure 12), leads to the typical double-hump profile form. Within the framework of the proposed conception it is not difficult to explain similarly the profile forms of other frequency harmonics observed in the valley position.

Thus, as was shown, many of the important properties of the oscilloscope traces, of phase and amplitude profiles of the frequency harmonics observed by Kachanov *et al.* (1984) as well as of the integral profiles observed by Klebanoff *et al.* (1962) can be qualitatively explained and understood from the viewpoint of the wave-resonance concept.

### 5.2. *z-Distributions*

The analysis of the nature of the K-regime of breakdown presented above can explain the causes of the appearance of the spikes in space observed by Kachanov *et al.* (1984).

As was noted in §4.2, a pair of harmonics $\pm\beta_l$ having the same frequency gives, by addition, a beating of the amplitudes in the $z$-direction and jumps in the phases of $\pi$, just as in the case of a standing wave. But the positions of the phase jumps

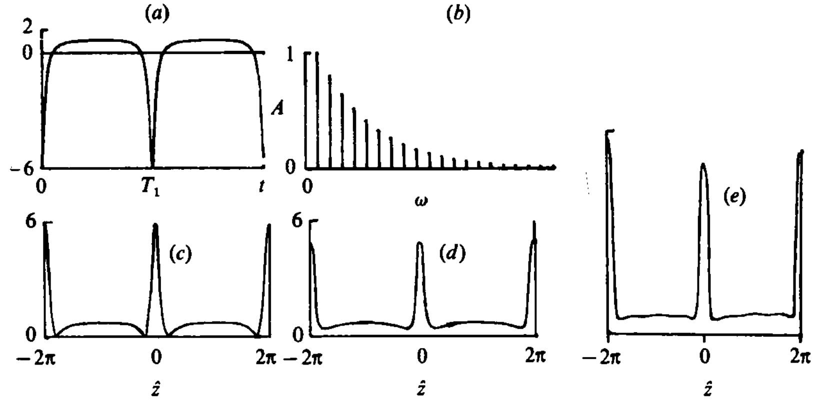


FIGURE 14. (*a–d*) Simulation of (*e*), the transverse distribution of pulsation amplitude obtained in Kachanov *et al.* (1984).

$z = (\frac{1}{2}\pi \pm k\pi)/\beta_l$, depend on the $\beta_l$. It is easy to show that the phase will remain equal to zero at the points $z = 0 \pm 2\pi/\beta_l$ (i.e. in the peak position) when $\beta_l$ is growing, and the amplitude will have its maximum as before. At the same time, at other points the amplitudes and the phases will change with $\beta_l$. Synchronization of the frequency–wavelength-harmonic crests at peak positions and lack of phase coordination at other points are quite analogous to the phase synchronization in time for the frequency harmonic, examined in §4.2. Therefore, when the wavelength spectrum contains enough harmonics for each frequency (as happens at rather large $x$), the distributions of the frequency-harmonic amplitudes in the $z$-direction will contain 'spikes in space'. They differ from the usual spikes in time because the change of sign is not important for the standing wave. Therefore, the absolute value of the oscilloscope trace with spikes should coincide qualitatively with the $z$-distribution of spikes. A model of the trace with spikes mentioned above and its absolute value are shown in figure 14 (*a*, *c*). The amplitude spectrum is shown in figure 14 (*b*) and corresponds to the position $x = 430$ mm in the experiments of Kachanov *et al.* (1984). The finite length of the hot wire in the $z$-direction (the length was equal to 0.5 mm and a typical spike width was about 2 mm) was taken into account in the curve shown in figure 14 (*d*), which is more smooth. This curve reproduces quite accurately the $z$-distribution of the fundamental frequency harmonic obtained in Kachanov *et al.* (1984) and shown in figure 14 (*e*).

The above picture of the superposition of standing waves having different $\beta_l$ also explains the phase distribution for all the frequency harmonics in the $z$-direction (figure 2*c*). In particular, it explains the shelf observed in the spike region which is a characteristic peculiarity of these distributions.

*Thus, the wave-resonance concept proposed in the present work can explain many of the important peculiarities of the behaviour of the disturbance amplitudes and phases observed in the four-dimensional space* $(x, y, z, t)$ *in experiments, both in the* K*-regime and in the* N*-regime of breakdown, as well as in the intermediate cases (see the work of Saric, Kozlov & Levchenko 1984).* In particular, it is easy to explain the transformation of the $\Lambda$-vortex structure under the influence of the subharmonic excitation observed in this work.

The existence of the phenomenon of the 'preferred period in the $z$-direction' and its cause has been discussed for more than 25 years. The proposed wave-resonance concept of breakdown allows us to solve this problem.

The $\Lambda$-vortices within the framework of this concept are the usual wave packets consisting of a lattice of frequency–wavelength harmonics that are synchronized in phase. As was shown in §§4.2 and 5.1, the chosen periodicity in the $z$-direction excited by the experimental spacers and based on the wavenumber $\beta_1$ does not strongly influence the process. From the wave-resonance point of view there are no conditions or limitations, apart from uniqueness imposed on the value of $\beta_1$. It is necessary that two or three harmonics $\beta_l = l\beta_1$ lie within the range of wavenumbers $\Delta\beta_s$ amplified by parametric resonances (i.e. the condition $\beta_1 \lesssim (2\text{–}3)\,\Delta\beta_s$ should be satisfied); otherwise the resonances will not have the coherent priming waves which are necessary for the amplification and formation of spikes. In the latter case the breakdown will be essentially different and either an amplification of the non-controlled background will take place (as in the experiments by Klebanoff & Tidstrom (1959) and Nishioka *et al.* (1980) carried out without the spacers) or the transition will switch over to the N-regime of breakdown (see I and II) if the fundamental wave amplitude or non-uniformities in the $z$-direction are not so large.

It follows from the wave-packet properties (see §3.1) that a decrease of the $\beta_1$ leads only to an increase of the distance between the packets (i.e. between $\Lambda$-vortices) in space but the packet width, the characteristic frequency of the oscillations within them, and even their form remain practically the same. These characteristics depend on the $\Delta\beta_s$ and $\beta_s$ (resonant wavenumber) which are controlled by the properties of the parametric resonance, the primary mean flow, the fundamental wave frequency, its intensity etc.

Such, in principle, is the solution of the problem of the preferred period in $z$. A definite period simply does not exist. The observed constancy of a typical $\lambda_{1z}$ within each different set of experiments can be easily explained within the framework of the wave-resonance concept through the initial wavelength spectrum which is set by the experimental conditions.

As far as the problem of the spike appearance is concerned, the question arises: how does the proposed wave-resonance concept correlate with the well-known local high-frequency secondary instability one? What do the results of the direct test of the latter concept obtained by Nishioka *et al.* (1980) mean from the new, wave-resonance viewpoint? Section 6 is devoted to these questions.

## 6. Two concepts of spike appearance

### 6.1. *The concept of local high-frequency secondary instability of a flow*

As is well known, the local high-frequency secondary-instability concept is based on the idea that the flashes of high-frequency oscillations, appearing locally in space and time with the period of the primary wave, are generated as a result of the instability of instantaneous flow velocity profiles formed by the primary wave in definite places. This concept was studied theoretically by Betchov (1960), Greenspan & Benney (1963), Gertsenshtein (1969), Landahl (1972), Zhigulyov *et al.* (1976), Zelman (1981), Itoh (1981) and others. In practice these instantaneous profiles are not formed by a sinusoidal wave. This primary wave is always strongly disturbed by the high harmonics and it is strongly modulated in the $z$-direction. The local high-frequency secondary-instability concept supposes that high-frequency oscillations observed at definite time moments in the region of the most deformed profiles (at the peak position) are connected with the amplification of small background waves with frequencies $\sim 10\omega_1$ as a result of local instability of the flow. There, an equation of the Orr–Sommerfeld type for the most unstable instantaneous inflexional profile is usually solved. (See for example the work of Nishioka *et al.* 1980.) Of course, it is assumed that the dependence of the real profile on $x$, $z$, $t$ slowly influences the process of the high-frequency-oscillation amplification. In accordance with Landahl's (1972) concept, the condition of equality of the high-frequency-oscillation-packet group velocity and the primary-wave phase velocity restricts the amplified high-frequency oscillations to the zone of unstable instantaneous velocity profiles for a long period and promotes the realization of a local high-frequency secondary instability, even if the increments are not very high.

Of course, the local high-frequency secondary-instability concept, local by its nature, cannot (and does not try to) explain the appearance in the peak position of the instantaneous velocity profiles, while the wave-resonance concept does. It also does not operate with the distributions of amplitudes and phases of the disturbances at different time moments and space locations (except for the moments and locations where spikes appear). The local high-frequency secondary-instability concept had been created only to explain the appearance of the flashes, in particular of the spikes, which take place against a background of the existing unstable low-frequency flow. An attempt to explain the formation of the inflexion profiles through the simple superposition of the mean flow and a fundamental wave of finite amplitude was made by Landahl (1972) and Craik (1980). The attempt by Craik (1980) is very close in spirit to the spectral representation discussed here, but (due to its simplicity) does not represent this phenomenon in detail.

For the problem of the appearance of the flashes (in the peak position, locally in time), the local high-frequency secondary-instability and wave-resonance concepts can be compared with each other. It is not clear *a priori* whether these two concepts are contradictory to one another or are mutually complementary, representing two different viewpoints of the same phenomenon, both of which are right. These two approaches seem at first sight so different that it might be thought easy to check their correlation experimentally; this is not so.

With the help of a simple example one may demonstrate the 'dualism' of possible explanations of typical experiments in the spirit of the work of Nishioka *et al.* (1980). Let us consider a simple model situation: local high-frequency secondary instability of the primary harmonic wave with amplitude $A_1$, frequency $\omega_1$ and initial phase $\theta_1$ is investigated relative to the high-frequency wave $A_2 \ll A_1$ with frequency $\omega_2 = 8.2\omega_1$ and initial phase $\theta_2$. From the spectral point of view the weakly nonlinear interaction of these waves will lead at leading order to the generation of harmonics with frequencies $\omega_2 \pm \omega_1$, i.e. $7.2\omega_1$ and $9.2\omega_1$ (figure 15*b*). The trace corresponding to this spectrum is shown in figure 15(*a*). The phases $\theta_k$ correspond to the combination interaction: $\theta_7 = \theta_2 - \theta_1$; $\theta_9 = \theta_2 + \theta_1$, where $\theta_1 = 0.9\pi$, $\theta_2 = -0.4\,\pi$ and are shown in figure 15(*c*). The flashes that take place in figure 15(*a*) are connected with the periodic synchronization of the harmonic phases (see §3.2) provided by the combination nonlinear interaction. It is seen that in this case the oscillation phase within the flash drifts in time. This is particularly clearly seen in figure 15(*d*) where five periods of the oscillations are superposed.

As seen from figure 15, the combination generation of the harmonics in the framework of the wave-resonance model looks very much like the local high-frequency secondary instability.

As already noted, the concept of the local high-frequency secondary instability of shear flows became widespread after the 1960s mainly owing to the experiments of Klebanoff *et al.* (1962) and Kovasznay *et al.* (1962). However, the appearance of many different theoretical models (quoted above) was not accompanied by further

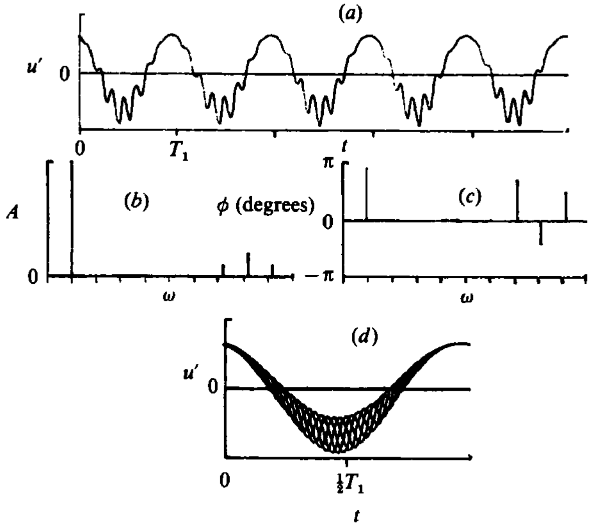


FIGURE 15. Simulation of nonlinear combination interaction.

experimental substantiation. The theories were based mainly on the above-mentioned experimental data.

The results of the first (and up to now the only) direct experimental investigations of the local high-frequency secondary-instability model were published by Nishioka *et al.* (1980), in an investigation of plane channel flow. The excited primary wave has a large amplitude and leads to the K-regime of breakdown. Somewhat downstream, a spatial packet of time-periodic high-frequency pulsations was introduced with the help of a point source. The investigations of the development of these oscillations downstream, in the region of inflexion instantaneous velocity profiles formed by the primary wave, should have given information about the local high-frequency secondary instability of the flow.

The main conclusion of Nishioka *et al.* (1980) is: '...we may say that the validity of the concept of secondary instability has been conclusively verified by the present investigation'. An analysis of the data obtained in their work shows that practically all of the main results correlate with the local high-frequency secondary-instability concept, but on the other hand, practically all of those results (including the peculiar properties of amplitude and phase distributions) can be also explained within the framework of the wave-resonance concept presented above!

### 6.2. *Comparison with observations of shear flows*

Let us now consider in detail some of the results obtained by Nishioka *et al.* (1980) and evaluate them from both points of view.

An amplification of the packet of high-frequency harmonics with the formation in the spectrum of a hillock located in the region of the same characteristic frequency is one of the necessary manifestations of local high-frequency secondary instability. The first visual estimate of this typical frequency was made by Klebanoff *et al.* (1962). Typical frequencies were greater than the fundamental one by approximately an order of magnitude. An estimation of the characteristic frequency has been carried out also in the experiments of Nishioka *et al.* (1980), but there a filter of high frequencies was used. Such a filter can itself create the previously mentioned hillock (!)

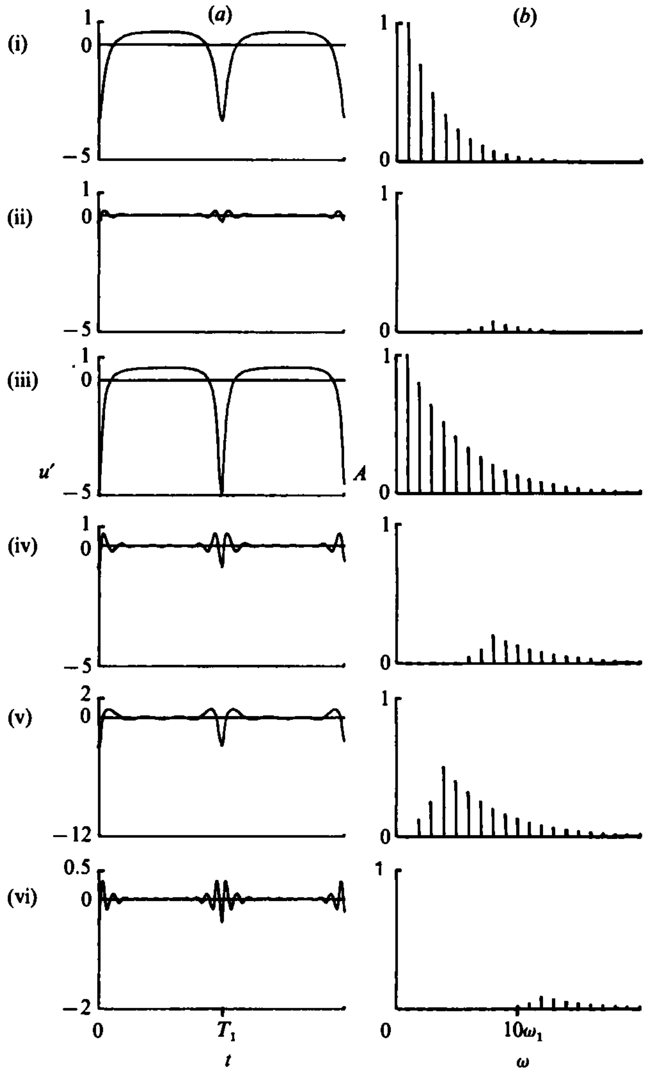


FIGURE 16. The influence of cutoff filter frequency on output signal interpreted as secondary disturbances.

in the required range of the high-frequency oscillations because in Nishioka *et al.* (1980) 'The cut-off frequency was set at a frequency 100 to 150 Hz lower than that of the HFP to be observed'. The spectra of pulsations, which simulate the disturbances at $x \approx 415$ and 430 mm in the peak position in the work of Kachanov *et al.* (1984), are shown in figure 16 together with the corresponding traces (graphs (i) and (iii)). With a filtered output one obtains the traces and spectra shown in graphs (ii) and (iv) (figure 16). In Nishioka *et al.* (1980) it was noted that '...the frequency of the natural HFP is about 500 Hz, which may be predicted from the controlled experiment'. But it is clear that this characteristic frequency depends *completely* on the cutoff frequency of the filter, because it corresponds to the mean frequency of the 'tail' in the spectrum which is left after filtration (see §3.1). Graphs (v) and (vi) in figure 16 differ from graphs (iv) only in the cutoff frequency. It is seen that the characteristic frequency of the flash observed changes too. For graphs (ii), (iv), (v) and (vi) the cutoff frequencies are about $7\omega_1$, $7\omega_1$, $3\omega_1$ and $11\omega_1$ respectively. (The fundamental frequencies in Kachanov *et al.* (1984) and Nishioka *et al.* (1980) were equal to 96.4 and 72 Hz respectively.)

Thus, one can obtain almost any characteristic frequency of the flashes within the wide natural spectrum. In view of this, the question is raised: where is the boundary between the primary and secondary (or low- and high-frequency) disturbances in the monotonically attenuating spectra that are observed in K-breakdown (see figure 16 or 8*a*) and what is the 'characteristic frequency' of the secondary disturbances here?

The local high-frequency secondary-instability model should provide an answer to this question. One hopes that it is possible, but it is not simple. This question does not exist, however, within the framework of the wave-resonance concept. At the same time the latter model is restricted only by the case of monotonic attenuation of harmonic amplitudes with frequency. In other, strongly nonlinear cases, when the hillock in the high-frequency spectrum occurs only the local high-frequency secondary-instability model (of the two mentioned) can be applied.

Analysis of the disturbance travelling velocities plays an important role in demonstrating the validity of the local high-frequency secondary instability concept. Figure 6 of Nishioka *et al.* (1980) demonstrates local shear-layer acceleration. At the same time, this phenomenon, i.e. the hillock observed in figure 6 of Nishioka *et al.* (1980) in the high-shear layer, is well explained in the framework of the wave-resonance concept and it correlates with the results of Kachanov *et al.* (1984). Though these results were obtained for boundary-layer flow, the absence in Nishioka *et al.* (1980) of the corresponding profiles makes us turn to the data of Kachanov *et al.* (1984). The striking coincidence of most of the results and the existence of very similar K-regimes of breakdown in both flows are solid reasons for doing so.

The point is that, in the region of the high-shear layer (i.e. in the region of spike appearance) the amplitudes of high harmonics have values close to their maxima (in $y$-profiles). At the same time, the amplitudes of the fundamental wave and its second harmonic have their maxima closer to the wall (see figure 13 and the explanation in §5.1). As a result, the 'weight' of the high harmonics in the integral signal analysed by Nishioka *et al.* (1980) becomes very large locally in $y$. These high harmonics, because of the synchronization of their valleys, give the slow peak orientated downwards that occurs in the region of the valleys of the integral oscillations. Locally in time (and near the valley) they impose their phase on the integral oscillations, but they interfere in other time moments and do not then contribute to the integral signal. At these other time moments and other $y$-coordinates, the fundamental wave distorted by its second harmonic predominates in the integral signals. This is why the increase, observed in Kachanov *et al.* (1984), of the high-frequency phase velocities, and the corresponding increase of the velocity of the spike formed by these harmonics (see figure 18 in Kachanov *et al.* (1984)), leads to acceleration of the characteristic points of the oscilloscope traces (which takes place in figure 6 in Nishioka *et al.* (1980) locally in $t$ and $y$) in the regions where the high harmonics 'set the tone'. In other regions, the 'phase velocities' (in the terminology of Nishioka *et al.* 1980) become almost constant in accordance with the behaviour of the phases of the fundamental wave prevailing here and its second harmonic.

So, in this case the wave-resonance concept provides an explanation of some initial conditions before the local high-frequency secondary instability appears, and it complements the latter model.

Figure 12 in Nishioka *et al.* (1980) demonstrated that the high-frequency-oscillation 'phase velocities' do not depend on their frequency, grow with the amplitude and are equal to the high-shear-layer velocity, which also grows downstream. Results close to these were obtained in Kachanov *et al.* (1984), where the phase velocities of high harmonics of the fundamental wave grow together with the group velocity of the spikes. From the local high-frequency secondary-instability point of view, figure 12 in Nishioka *et al.* (1980) is evidence of the satisfaction of Landahl's (1972) criterion ones. Unfortunately, there is no information in Nishioka *et al.* (1980) about the

But it is necessary to note one essential circumstance which was noted first in Kachanov *et al.* (1984). Namely, the *usually determined* inflexion instantaneous profiles on the one hand and the oscilloscope traces, particularly those showing spikes, on the other hand are one and the same phenomenon! These are just two different sections of the same set of experimental data, respectively obtained at $t$ = const and $y$ = const, at each fixed $x$ and $z$. The spike velocity is here always exactly equal to the high-shear-layer speed, because the high-shear layer *is* the spike, not its *cause*.

To verify whether Landahl's criterion is satisfied, it is first necessary to demarcate in the frequency spectrum the regions of a primary (low-frequency) and a secondary (high-frequency) oscillation. Then it will be possible to compare the velocity of the shear layer formed *only* by primary oscillations with the group velocity of secondary ones. Unfortunately, there is no information in Nishioka *et al.* (1980) about the method of determination of the instantaneous velocity profiles. The high-shear-layer speed was not determined in Kachanov *et al.* (1984) at all; the attention of the authors was concentrated on the group speed of the high-frequency oscillations and the phase speed of the fundamental wave and its higher harmonics. From the wave-resonance point of view, the satisfaction of Landahl's criterion can be shown to be equivalent to the phase synchronization phenomenon, which is an inherent property of parametric resonance (see §4.2).

The profiles of the $y$-distributions of the high-frequency-oscillation amplitudes shown in figures 9 and 14 of Nishioka *et al.* (1980) are in a good qualitative agreement with the high-frequency-harmonic profiles obtained in Kachanov *et al.* (1984) (see figure 13). The integral high-frequency oscillation consists of these high harmonics. The form of such profiles has been well modelled in §5.1 by the simple superposition of a set of spatial waves for each of the frequencies, and each has the form of profile typical of the linear region of the development. The change in time of the flash amplitude, is connected, as was noted above, with the synchronization of the harmonic phases which takes place near the time moments (*b*) and (*c*) of Nishioka *et al.*'s figure 9.

To demonstrate the comparability of the results of Nishioka *et al.* (1980) with the wave-resonance concept and to show the dualism, mentioned above, of wave-resonance and local high-frequency secondary-instability representations (as concerns the problem of flash appearance), two pairs of traces are shown in figure 17 (graphs i). The first pair (*a*) was obtained with the help of a computer simulation of combination wave interaction in the framework of the wave-resonance concept under the conditions of the experiment in Nishioka *et al.* (1980). The second curve (*b*) is a reproduction of figure 8 from Nishioka *et al.* (1980). The amplitude spectra of the upper and lower (after the filter) curves (*a*)(i) are shown in figure 17 (*a*) (curves ii and iii). The low-frequency part of the spectrum (*a*)(ii) corresponds to the attenuation of harmonic amplitudes in accordance with a geometric progression with factor $q = 0.4$ and models the primary low-frequency pulsations in Nishioka *et al.* (1980) ($\omega_1 = 72$ Hz). The amplitudes of high-frequency harmonics in spectra (*a*)(ii) and (*b*)(ii) simulate the reproduction, by combination interaction, of a priming

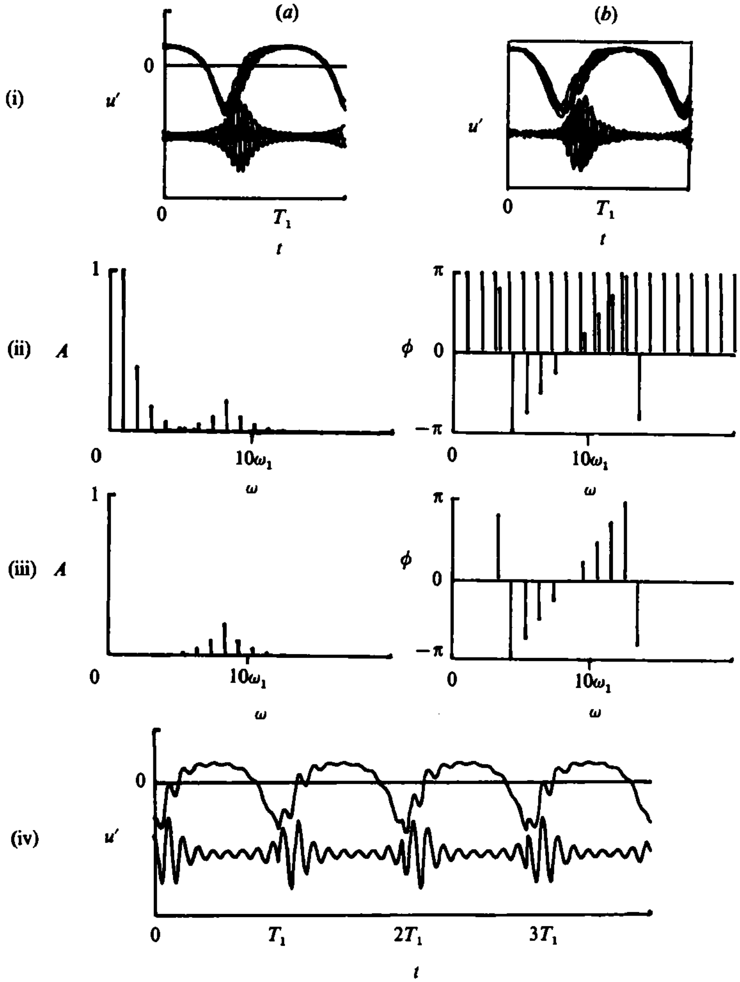


FIGURE 17. Oscilloscope traces from the experiments by Nishioka *et al.* (1980) (curve *b*(i)) and its simulation within the framework of the wave-resonance concept (curves *a* (i) and (*a*, *b*) (ii), (iii), (iv)).

high-frequency wave with $\omega_p = 600$ Hz, which was introduced in the flow in Nishioka *et al.* (1980). Note that although the actual values of high-frequency-pulsation amplitudes were slightly varied to coincide with the experiment, the phases of high-frequency harmonics were obtained only from the properties of combination interaction. The curves (i) (in figure 17*b*) are presented for the unified timescale in graph (iv), figure 17.

A comparison of the curves (*a*) and (*b*) in figure 17 illustrates the fact that the appearance of a high-frequency wave packet, observed in Nishioka *et al.* (1980) under high-frequency excitation, can be considered from both the local high-frequency secondary-instability and wave-resonance points of view. It does not mean that the development of high-frequency oscillations, under the conditions of the controlled experiments for the investigations of local high-frequency secondary instability such as in Nishioka *et al.* (1980), is limited by a combination interaction. It is only the first (fastest) step of wave development. Then the conditions appear for the parametric amplification of priming harmonic waves with frequencies $\omega_k \lesssim \omega_p$ (see §4). It can be shown that the parametric resonant amplification explains qualitatively the phenomenon of the fast decrease of a typical frequency within the high-frequency-oscillation packet which was observed in Nishioka *et al.* (1980). It is connected with the growth of the weight of relatively low-frequency pulsations within the secondary high-frequency-oscillation packet, which takes place as a result of a parametric amplification of a broad spectrum of 'subharmonic' waves (see §3). This phenomenon can probably be explained in the framework of the local high-frequency-instability concept too (for example, through the downstream transformation of the instantaneous velocity profile), although the explanation presented in Nishioka *et al.* (1980) (through the wave-packet acceleration) is not correct.

## 7. Conclusion

The wave-resonance idea presented here does not constitute a complete theory only the wave-resonance concept exists now. To decide conclusively the question of the place and role of this concept in understanding laminar-boundary-layer breakdown, it is necessary to carry out real calculations and compare the results with experiments. This may be direct numerical computations like those by Kleiser (1982), Orszag & Patera (1983), Wray & Hussaini (1984), and Laurien (1986) or some analytical results in the spirit of the Craik–Nayfeh–Herbert–Zelman studies, which of course pose great difficulties but are possible in principle. As with the results of computer experiments, they should be analysed, just as experimental data, to determine the principal mechanisms that are responsible for a breakdown. It is not sufficient to obtain good agreement between calculation results and experimental ones, it is necessary to understand why these results are obtained.

The proposed wave-resonance concept is just such an attempt to understand the mechanisms that dominate the first stages of breakdown, including the appearance of spikes. The wave-resonance model, even at the present conceptual level has great potential to explain (at least qualitatively) many observed features of laminar-boundary-layer breakdown in both known regimes. The analysis of the N-regime of breakdown was carried out by Kachanov & Levchenko (1982); the K-regime has been discussed in the present work. It is not difficult to show that in the framework of the wave-resonance concept many properties of intermediate regimes and the transformation of a transition process from the N- to the K-regime (and vice versa), studied by Saric *et al.* (1984), can also be better understood.

It should be noted that the parametric resonant amplification of background random subharmonic priming waves, which leads to randomization in the N-regime of breakdown, can also take place in the K-regime. Moreover, the appearance in the K-regime of a large number of higher harmonics (including the plane ones) multiplies this process. It can be observed to take place in the K-regime in the latest stages of the development. That is why the name 'subharmonic regime' is non-descriptive and the term 'N-regime' is more short, convenient and preferable. Note that the terms 'C-type' and 'H-type' of secondary instability introduced by Saric *et al.* (1984) should not be confused with the types (regimes) of breakdown; because, in contrast with the Craik and Herbert theories, there is no convincing evidence of a difference between these regimes, both of which correspond to the N-breakdown first observed in I.

As far as the process of the local appearance in space and time of spikes is concerned, the wave-resonance concept complements the well-known local high-frequency secondary-instability concept and provides a new viewpoint that can give some additional insight, especially after construction of the corresponding theory. In particular, most of experimental results by Nishioka *et al.* (1980) (where the local high-frequency secondary instability concept was substantiated) can be qualitatively explained in the framework of the wave-resonance concept. At the same time, this concept operates not only with the local process of flash appearance, but also claims to give an explanation of the formation of the inflexional instantaneous profiles as well as of the structure of the whole field of the disturbances (beginning almost from their sources and including the formation of spikes and their doubling both in the peak and valley positions) in four-dimensional space $(x, y, z, t)$. The analysis of the applicability of the wave-resonance concept to the description of these phenomena, carried out in this work, is very plausible.

One can conclude that the development of the wave-resonance concept and its application to the analysis of numerical and physical experiments, together with the construction on this basis of a complete theory, can give an essential impetus towards a better understanding of the nature of breakdown.

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